Making Higher Mathematics Accessible: G.A. Miller’s Project for a Mathematical Dictionary

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CIRMATH-Americas
University of Virginia, 27-30 May 2018
American mathematicians have good reason to protest against such a butchery of their subject in a popular work of reference.

Mathematical Definitions in the New Standard Dictionary, 1913
George Abram Miller (1863–1951)

I have heard the question raised as to where or when our American LaGranges are to appear.

A Decade of American Mathematics, 1921
Oliver Dimon Kellogg (1878–1932)

[...] mathematicians are still in the shadow of the Tower of Babel [...]
Conference Goals:

▶ To provide a venue in which European CIRMATH members can intensively interact with historians of mathematics from the Americas working on related historical questions.

▶ To provide researchers the opportunity to explore the significant archival and library holdings at both the University of Virginia and the nearby Library of Congress.

▶ To provide opportunities for American and European researchers at earlier stages in their careers to develop significant and enduring research collaborations with established scholars in the history of mathematics.

▶ To trace the impact of journals on the circulation of mathematics within the Americas and between the Americas and Europe from 1850 to 1950.
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Introduction

1894

The American Mathematical Monthly.


1915

Mathematical Association of America founded. The charter members of the MAA—those who joined by April 1916—numbered 1,046. The largest number of members came from New York (120), followed by Illinois (97), Ohio (72), Massachusetts (70), and Pennsylvania (67).

1916

Earle Raymond Hedrick, first MAA president.

First MAA section: Ohio.

First summer meeting: MIT. September 1–2.
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1916
Second summer meeting of the MAA, September 1917:

A plan for a mathematical dictionary, suggested in the main by Professor G.A. Miller, was brought by him before the Council.
Remediating technical errors and historical accuracies:

Miller:

[...] recognized that an historian who disapproves of all the available histories of mathematics is under some obligation to write a history of his own.
Miller on Dodd, Mead, and Co.’s *New International Encyclopedia*:

*Riemann and Helmholtz formulated assumptions for a geometry [...] with constant curvature and observed that on the sphere, whose curvature is constant and positive, the sum of the angles of a triangle is less than a straight angle [...]”

*Abel, by the use of the theory of groups discovered by Galois, gave the first satisfactory proof of the fact, anticipated by Gauss and announced by Ruffini, that it is impossible to express the solution of a general equation as algebraic functions of the coefficients when the degree exceeds the fourth.*

*The few modifications which have been suggested could scarcely be supposed to be of general interest to mathematics teachers if they did not relate to an excellent work of reference which is very extensively used by college and university students. It is evidently highly desirable that such works be as clear and accurate as possible.*
Miller on Davies and Peck’s *Mathematical Dictionary and Cyclopedia of Mathematical Science*:

*The present article was suggested by extravagant claims made in the advertisements of some general works of reference*. As an extreme case we may refer to recent advertisements of the *Mathematical Dictionary and Cyclopedia of Mathematical Science* by Charles Davies and W.G. Peck, which contained the following statements: “A standard work for 60 years”. “Definitions of all terms employed in mathematics, an analysis of each branch and the whole as forming a single science.”

*The claims represented by these quotations, which many readers of this Monthly have doubtless seen, present a serious alight to our intelligence* […]

— Miller’s *The Obsolete in Mathematics*, 1917
Miller on Funk and Wagnalls’ *New Standard Dictionary of the English Language*:

[...] not reliable as regards definitions of mathematical terms. [...] 

A still more original and more mysterious definition [...] relates to the regular group. [...] this is “a transitive group whose order is the same as that of the letter on which it is made.” It is very difficult to see how any one can discover any meaning whatever in such a definition. To make a group on a letter is a process which seems to have been foreign to the literature of this subject. A large number of almost equally vague statements occur under other terms. [...] an irrational number is “a definite number not expressible in a definite number of digits,” and a congruence group is defined as a group made up of replacements.

It may probably be assumed that all mathematicians who read these few citations will agree that American mathematicians have good reason to protest against such a butchery of their subject in a popular work of reference.
Norman Anning, University of Michigan, 1921:

[...] the eleventh edition of the Encyclopedia Britannica reveals, in one short paragraph, the near facts:

Analytically the hyperbola is given by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ wherein $ab > h^2$. In the rectangular hyperbola $a = b$; hence its equation is $x^2 - y^2 = 0$ ...

How can we believe and teach the doctrine that (as Bacon might have said it) “Mathematicks maketh the careful man” if such inaccurate statements are permitted in books which the public has a right to regard as sources of exact information?
On The Needs of Fledgling Scholars:

In view of the importance of the subject [...] it seems desirable to endeavor to aid the young reader by directing attention to its shortcomings, especially since such a reader is usually inclined to exercise too little caution in accepting results found in what are commonly regarded as standard works of reference.

— Miller on Dodd, Mead, and Co.’s New International Encyclopedia

The English-speaking pure mathematicians constitute more nearly a terra incognita than the workers in any other large field of knowledge. This is partly due [...] to the fact that there are so few mathematical works of reference in the English language. There never has been a good mathematical encyclopedia or other work of general reference in this language [...] As a result of this lack of intermediate mathematical literature comparatively few of our people know what constitutes a mathematician of high order.

— Miller’s American Mathematics, 1911
On The Needs of Fledgling Scholars:

Comprehensive views are often very helpful and fill an important place in mathematical education. The mathematical student needs to use the telescope as well as the microscope, even if the latter instrument is more important for those who desire to become experts along mathematical lines. The training which is best adapted to the needs of the future expect is, however, not always the most valuable for the average student.

— Miller’s Historical Introduction to Mathematical Literature, 1916

[...] aim at a more comprehensive knowledge of his subject than that possessed by those who are now in active service [....]

— Miller’s Suggestions for the Prospective Mathematician, 1914
Securing a consensus of opinion in the MAA, 1917:

It was agreed that the plan, if it can be carried out, is so important and that the questions of the various details, as to size, cost, extent, etc., are so involved, that a committee of five should be appointed by the president, to make an extended study of the plan, and to report to the Council.
Scope and style, 1918:

[...] brief definitions of the words employed up to and including the end of collegiate work proper, or possibly including the first graduate year.

[...] elementary algebra and geometry, trigonometry, plane and solid analytic geometry, and a first and second course in the calculus, together with elementary first courses in differential equations, higher analytic geometry (including curves and surfaces), higher algebra (including theory of equations, determinants, etc.), the theory of numbers, the theory of functions of real and of complex variables, the theory of groups, descriptive geometry, projective geometry, line geometry, differential geometry, mechanics, and the theory of probability and statistics.
Group (grōŏp), groupe, Gruppe, gruppno, gruupo. This term is frequently used in mathematics with its non-technical meaning, denoting a collection composed of a finite or of an infinite number of elements; e. g., a group of terms, a group of points, a group of waves, group insurance, etc. The most general technical mathematical meaning of the word group is obtained by restricting its non-technical meaning by imposing the condition that each of the elements of the collection can be combined with itself and also with every other element of the collection. Moreover, the result obtained by such a combination must also be an element of the collection. This most general technical meaning of the term group includes all its other technical meanings as well as that of domain of rationality. It appears frequently in the geometrical literature, but it occurs also in the literature on analysis. Cf. Encyclopédie des Sciences Mathématiques, tome 1, volume 2, page 248.

The most common additional restriction imposed on the elements of a group is that the inverse of each element shall also be contained in the collection; i.e., each element of the collection can be combined with at least one element of the collection so as to obtain the identity. Among the most common additional restrictions are the following: when the elements are combined, or multiplied together, the associative law holds, and if any two of the symbols in the equation

\[ xy = z \]

the inverse of any one of its elements is discussed. Some writers contend that even an infinite collection does not represent a group unless its elements satisfy all the conditions of combination noted above. Cf. A. Loewy, Archiv der Mathematik und Physik, volume 9, (1905), page 105, where it is stated that the definition given on page 218, volume 1, Encyclopädie der Mathematischen Wissenschaften, is incorrect as regards infinite groups, because it is satisfied by a collection of elements which does not include the inverses of its elements. A group in the most restrictive sense of the term has been called ordinary group by L. Autonne, Paris, Comptes Rendus, vol. 143 (1906), p. 671.

E. Galois (1811–1832) seems to have first used the term group with a technical mathematical meaning. In fact, Galois and the other writers before the middle of the nineteenth century practically confined their group theory studies to finite groups whose elements are represented by substitutions, and every finite collection of substitutions satisfies the most restrictive definition of group noted above provided it satisfies the most general technical definition. Difficulties as regards the most desirable definitions of the technical term group began to appear when A. Cayley inaugurated the study of abstract groups (1854), and C. Jordan exhibited the wide usefulness of infinite groups (1868). These difficulties cannot be regarded as solved at the present time.

In a broad way groups have been divided
Statistics (stā-tīs’tiks), F. statistique, G. Statistik, I. Statistica, Sp. estadistica. The word statistics seems to be derived from the Latin status, used in the sense of a political state. Statistics is a comparatively new word. Its first occurrence in English thus far noted seems to be in J. F. von Bielfeld, The Elements of Universal Erudition, translated by W. Hooper, London, 1770. One of the chapters of this book is called Statistics, and the subject is defined as “The science that teaches us what is the political arrangement of all the modern states of the known world.” The word occurs in 1787 with a somewhat changed meaning in the preface to E. A. W. Zimmerman, A Political Survey of the Present State of Europe. In this work it is stated that about forty years before the branch known as statistics was formed into a separate science in Germany. The German word statistik was used by Professor Achenwall of Göttingen in 1749, and the Latin, statisticus, was used at a somewhat earlier date. In Meyers Konversationslexikon, 6 ed., volume 18, under “Statistik”, Schäfer (1735–1809), a pupil of Achenwall, defined: “Statistik ist stillstehende Geschichte; Geschichte ist fortfallende Statistik.” In 1790 Sir John Sinclair stated in a letter to the Clergy of the Church of Scotland that “statistical inquiries” have been carried to a great extent in Germany, and adds that the expression “statistical inquiries” means “inquiries respecting the population, the political circumstances, the productions of a country, and other matters of state.” Statistics, as thus used by German writers and by others in the eighteenth century, meant an exposition of the character of the state, and such expositions were usually verbal rather than numerical. With the growth of official numerical data, it was natural that numerical statements should begin to replace the verbal statements. Statistics thus gradually came to mean an exposition of the attributes of the state by numerical methods. Following this usage, the word next came to denote the figures used in such descriptions. Thus, the collections of numerical data were called statistics. This use of the word prevails at the present time but the data may refer to the state or to any other subject.

There is, however, an element in the meaning of the word statistics as used at the present time in the theory of statistics that is not necessarily involved in a collection of figures. Thus, a collection of 1000 numbers consisting
Doubts Raised, 1919:

The chief questions of doubt raised in the discussion were (1) whether such work would interfere seriously with the output of research; (2) whether some larger project should not be undertaken, such, for example, as the founding of a journal for abstracts in the English language to fill the place formerly occupied by the Fortschritle. […]

Professor [Roland George] Richardson […] stated that we should attempt something at once to replace the Fortschritle, and that this would not be possible if we were to attempt the dictionary […]

Professor [Ernest William] Brown stated that, but he finds […] business difficulties, particularly as to the relative importance of this project and the plan to institute a proposed journal to replace the Fortschritle and an Anglo-American or an English encyclopedia.
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Professor [Ernest William] Brown stated that he would undoubtedly order three copies of the dictionary for himself and the libraries, but he finds […] business difficulties, particularly as to the relative importance of this project and the plan to institute a proposed journal to replace the Fortschrritte and an Anglo-American or an English encyclopedia.
No project is ever endorsed unanimously. The fact that the Dictionary is so generally approved seems to me a proof that the idea is sound. Nor do I fear any dissipation of mathematical energy in the preparation of the manuscript. There are enough well equipped men not putting their time mainly on research to do the work without lessening the research output. As there is no other big mathematical project afoot, it seems to me that there is nothing to lose in this Dictionary project, while I am one of those who strongly believe that there is considerable serious value to gain in general and that the pedagogical value is undisputed.
Service to Students, 1918–1919:

Herbert Ellsworth Slaught, University of Chicago:

[...] the only justification [...] is the real service which it may render in the collegiate field where at present no reference books comparable in scope and potential usefulness are available.

Aloysius Frumveller, Marquette University:

That such a dictionary is greatly needed by those who are not within reach of large mathematical libraries, as is the case with most teachers and students of mathematics in the colleges of the country.

Leonard Eugene Dickson, University of Chicago:

I have been convinced [...] that it would prove to be a valuable aid to mathematics, partly to experts when reading outside their specialties, but mainly to amateurs, to students, and to persons early in their mathematical careers. I believe many overlook the great extent of technical mathematical language in the literature and the fact that difficulties of students are very largely due to their failure to understand this technical language. [...]
Service to Scholars:

Science is a unit and it has a common language. The confusion of tongues is one of our grave dangers and the lesson of the tower of Babel demands our serious attention.

— Miller, Remarks to the New Members of the Illinois Chapter [of Sigma Xi], 1913

The extent to which new mathematical terms have been introduced has been viewed by some with considerable misgivings. Already in 1869 H. Hankel was led to compare mathematics with the Tower of Babel in view of the fact that mathematicians are speaking such different languages.

— Miller, Historical Introduction to Mathematical Literature, 1916

[...] mathematicians had great difficulty in understanding each other on account of their numerous special terms with continually widening meanings. This situation has grown much worse during the last half-century in spite of various mitigating movements.

— Miller, Mathematics. In: The American Yearbook, 1918
Service to Scholars, 1919:

Raymond Clare Archibald, Brown University:

[...] the order and extent of inclusion of foreign languages should be French, German, Italian, Swedish, Dutch, and possibly Russian or Spanish.

J.B. Shaw, University of Illinois:

[...] at the University of Illinois [...] the staff coöperate in keeping the run of mathematical journals, each one reporting on the matter found in some particular journal. Incidental to such reading as this, one often needs a mathematical dictionary when reading in a field to one side of his own special field.

Edward V. Huntington, Harvard University:

[...] instanced the attitude which the non-mathematician often holds regarding mathematics, quoting a dismayed reviewer of a book on statistics, who said that the technical terms in whose scientific exactness mathematicians pride themselves are forbidding and impossible. A mathematical dictionary would help such outsiders.
Reflecting on America, Past and Present:

As [Davies and Peck’s Mathematical Dictionary and Cyclopedia of Mathematical Science] was issued before American mathematicians were keeping abreast the advances made in Europe, [...] it reflects a status of mathematical knowledge which is too old to be of much interest to the modern student.

— Miller, Mathematics. In: The American Yearbook, 1918

Such names as those of Euler, Cauchy, Gauss, Lagrange, Galois, Abel and Cayley have no equals in the history of American mathematics; and [...] there are no American names which should be placed on a mathematical equality with those of Poincaré, Klein, Hilbert, Frobenius, Jordan, Picard and Darboux. [...]

— Miller, American Mathematics, 1911
The Value to American Mathematics of a Dictionary:

Oliver Dimon Kellogg, retiring VP of Section A [math] of AAAS, 1920:

*I have heard the question raised as to where or when our American LaGranges are to appear. There is little doubt but that young men of a high degree of genius exist in this country in every generation. That more of them do not find their way into mathematics, or, having found their way, do not continue on and develop there, is, to a considerable extent, a matter of environment.*

[...] *a desire for national preeminence in scientific attainment is most wholesome and valuable. I wish I might, therefore, compare the work of America during the decade with that of other countries. [...]*

*Titles of American books in the lists of current publications in higher mathematics are as needles in a haystack. The value to American mathematics of authoritative and up to date handbooks by American authors has been sufficiently emphasized to need no further comment here.*
Shaping Images of American Mathematics

Anning, University of Michigan, 1921:

*The efforts of the Association to produce a Mathematical Dictionary that shall be a credit to American scholarship are, in the highest degree, praiseworthy.*

Third Annual Meeting of the MAA, 1918:

*It is felt that the venture is worth while only if it can be completed in a thoroughly scholarly manner, and so as to reflect credit upon the Association and upon American mathematics as a whole.*

David Eugene Smith, Columbia University, 1919:

*If we go into the matter, I feel that we should produce a work that will be a standard in Europe as well as America for many years to come, — one which will be an honor to American scholarship and to American book manufacturing.*
Reconciling Old and New Identities

Giulio Vivanti (1859–1949), 1916:

A national science is an absurdity and he would be foolish who would refuse a scientific truth because it arose from beyond the Alps or the sea; but the work of scientific exposition and publication can be and ought to be national.

Miller, *Scientific Activity and the War*, 1918:

The preparation of scholarly works of the highest possible order at the present time is thus seen to be a patriotic service, which should be considered very seriously by those who are in position to render it. […]

Our students should not have to feel that the great majority of the best expository works relating to their subject are to be found only in the language of a people of low ideals imbued with a morbid desire to dominate the world at any cost.
Reconciling Old and New Identities

Frederick John Teggart, AAUP Vice-Chairman of Committee on the Apparatus of Research (CAR), 1919:

*The Committee was appointed for the purpose of promoting the publication in this country of at least some part of that apparatus of scholarship which we have long been accepting as of necessity “made in Germany”.*

Fourth Summer Meeting of the MAA, 1919:

*[CAR . . .] stands ready to aid the dictionary project directly, if funds can be secured […]*
A Receptive Climate

Robert S. Woodward, former AMS president (1989–1900), 1916:

*When one looks back, as I am able to, a full half century over the development of mathematical studies in America, and when one reflects on the admirable progress of the past quarter of a century, he is stimulated to entertain hopes, if not glorious visions, for the future.*

Slaught, Retiring MAA Presidential Address, 1920:

*The lists of papers read before the [AMS] and subsequently published show an average annual output for the twenty years of 108 papers. These papers and numerous others [. . .], together with monographs and treatises published by educational foundations, by universities, or by commercial publishers, together constitute a scientific output in mathematics which not only makes a favorable showing for the past but also augurs well for the future.*
The New Opportunity for Science

Hedrick, 1917:

*Especially under the stress of war, public attention may be secured* for the real claim of mathematics as a public necessity.*

In the present emergency, popular conviction of the real significance of analytic geometry for society is being attained, and may be fostered, through the occurrence of just such graphical laws in the dynamics of airplanes, in artillery performance (ballistics) and in wireless telegraphy.

Robert A. Millikan*, 1919:

*But what have all these accomplishments of science in the war to do with the new opportunity in science? Simply this; for the first time in history the world has been waked up by the war to an appreciation of what science can do.*
The New Opportunity for Science

Robert A. Millikan*, 1919:

*Member of a 1919 NRC committee “to consider the proposal of a mathematical dictionary and report to the Executive Committee of the Division [of Physical Sciences]”. Included Slaught, Eliakim Hastings Moore, Leonard Eugene Dickson, and astronomer Forest Ray Moulton.
John Miller Burnam (1864–1921), Chairman of CAR, 1918:

[…] when the present mighty conflict is over, almost the only nation having much left will be the United States. This fact will impose on us the duty of supplying the university work for the remainder of the world, or rather we should regard this as a glorious opportunity to benefit mankind.

MAA Secretary-Treasurer William DeWeese Cairns, 1919:

The view was expressed that all such work, throughout the field called the Apparatus of Research, will devolve more and more upon America. This dictionary project, on account of its relative simplicity and on account of its natural terminability, offers a safe means of initiating such work in this country, to train men toward such work and to determine its feasibility here.

Raymond Clare Archibald, Brown University, 1919:

[…] of several important undertakings that demand intense coöperative effort on the part of mathematicians it is the dictionary project alone which we, as a nation, are equipped to carry through unaided.
A truly massive undertaking:

Archibald, 1919: $63,000 (four editors; 10-member consulting board)

Smith, 1919: $30,000

It is evident that a dictionary could be prepared for less money. If it cost half as much, I should think it would be worth about one tenth as much.

Slaught, 1922:

A lump sum of $100,000 or of $20,000 per year for five years, will be needed for the preparation of the manuscript. Such a work would be monumental in character and would insure great honor to any donor.

Parshall (2015): Equivalent to $6,180,000 in 2013 American dollars.
### Table: Estimated balances on business at the close of the indicated calendar year.

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General Endowment Fund (1917–1937): $18,200
Some Key Findings:

- Project hinged on financial generosity of donors; no MAA contributions ⇒ not prioritized by MAA (?)
  - Long-lasting and enthusiastic commitment to dictionary project largely symbolic

- Captures the tension between “isolationist” U.S. climate and cosmopolitan perspective on mathematics and mathematical publication

- Illuminates shared belief that America had a special opportunity and optimistic outlook after WWI:
  - Could use dictionary project to train Americans and test American capacity for leadership in mathematical publication
  - Could use dictionary project (splashy) to shape image of America abroad
Thanks!

Selected Bibliography: